B.Sc./1st Sem (H)/MATH/22 (CBCS)

2022

1st Semester Examination

MATHEMATICS (Honours)

Paper : C 1-T

[Calculus, Geometry and Differential Equation]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any ***ten*** questions : 2×10=20

(i) Find y_n for the function $y = \dfrac{x^n}{x-1}$.

(ii) Show that the curve $y^3 = 8x^2$ is concave to the foot of the ordinate everywhere except at the origin.

(iii) If the axes are rotated through an angle 45° without changing the origin, then find the new form of the equation $x^2 - y^2 = a^2$.

P.T.O.

(iv) Find the equation of the circle lying on the sphere $x^2 + y^2 + z^2 - 2y - 4z = 11$ and having its centre at (1,3,4).

(v) Find the total area of the circle $x^2 + y^2 + 2x = 9$.

(vi) If $I_n = \int_0^{\pi/4} \tan^n x\,dx$, for $n \geq 2$, find the value of $I_n + I_{n-2}$.

(vii) Find the asymptotes of the curve $x^3 + y^3 = 3axy$.

(viii) Find the integrating factor of $(1+x^2)y_1 + y = e^{\tan^{-1}x}$.

(ix) Find the singular solution of $y = x\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^2$.

(x) Find the nature of the conic $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$

(xi) Calculate the sum of the reciprocals of two perpendicular focal chord of the conic $\frac{l}{r} = 1 + e\cos\theta$.

(xii) Show that $\lim_{x\to\infty}\left(\frac{ax+1}{ax-1}\right)^x = e^{2/a}$, $a > 0$.

(xiii) If $u = \sin ax + \cos ax$, show that

$$u_n = a^n \left\{1 + (-1)^n \sin 2ax\right\}^{\frac{1}{2}}.$$

(xiv) Solve $p - \frac{1}{p} - \frac{x}{y} + \frac{y}{x} = 0$ where $p \equiv \frac{dy}{dx}$.

(xv) Evaluate $\lim_{x \to \infty} \left(\sqrt{x^2 + 2x} - x\right)$.

Group - B

2. Answer any ***four*** questions : 5×4=20

(i) State and prove Leibnitz's theorem. If $y = \tan^{-1} x$ find $(y_n)_0$ by using Leibnitz's theorem.

(ii) Prove that the locus of the middle points of focal chords of a conic is an another conic.

(iii) If $J_n = \int \sin n\theta \sec\theta d\theta$, show that

$J_n + J_{n-2} = -\frac{2}{n-1} \cos(n-1)\theta$. Hence deduce the value $\int_0^{\pi/2} \frac{\sin 3\theta \cos 3\theta}{\cos\theta} d\theta$.

(iv) If S be the length of the arc of $3ay^2 = x(x-a)^2$, measured from the origin to the point (x, y), show that $3s^2 = 4x^2 + 3y^2$.

P.T.O.

(v) Find the equation to the right circular cylinder of radius a, whose axis passes through the origins and makes equal angles with the co-ordinates axes.

(vi) Solve : $16x^2 + 2\left(\frac{dy}{dx}\right)^2 y - \left(\frac{dy}{dx}\right)^3 x = 0$.

Group - C

3. Answer any *two* questions : 10×2=20

(i) (a) Explain L'Hospital Rule. Using L'Hospital Rule prove that

$$\lim_{x\to\infty}\left[\frac{a_1^{1/x} + a_2^{1/x} + \ldots + a_n^{1/x}}{n}\right]^{nx} = a_1 a_2 \ldots a_n .$$

(b) Find the envelop of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, a and b are variable parameters connected by the relation $a + b = c$. 5+5

(ii) (a) What is a great circle ? Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as the great circle.

(b) Reduce the equation $3x^2 + 5y^2 + 3z^2 + 2yz + 2zx + 2xy - 4x - 8z + 5 = 0$, to the standard form and find the nature of the conic. 3+7

(iii) (a) Find the volume of ellipsoid generated by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about major axis and minor axis.

(b) Define singular and general solution of the differential equation. Find the both solutions of the following differential equation : $p^3x-p^2y-1=0$. 5+5

(iv) (a) Find the rectilinear asymptotes of the following curve :

$$x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0.$$

(b) If $f(m,n)=\int_0^{\pi/2}\cos^m x\sin nx\,dx$ prove that

$$f(m,n)=\frac{1}{m+n}+\frac{m}{m+n}f(m-1,n-1),$$

$m,n>0$. Hence deduce that

$$f(m,n)=\frac{1}{2^{m+1}}\left(\frac{2}{1}+\frac{2^2}{2}+\frac{2^3}{3}+\ldots+\frac{2^m}{m}\right).$$

5+5

2022

1st Semester Examination

MATHEMATICS (Honours)

Paper : C 2-T

[Algebra]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any ***ten*** questions : 2×10=20

(a) If a, b, c be three positive real numbers in Harmonic Progression and n be a positive integer greater than 1, then prove that $a^n + c^n = 2b^n$.

(b) Geometrically represent the complex number $z = a + bi$.

(c) Find the conditions that the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ are in G.P.

P.T.O.

(d) Apply Descartes' rule of signs to determine the nature of the roots of the equation $x^4 + x^2 + x - 1 = 0$.

(e) Diminish the roots of $4x^3 - 8x^2 - 19x + 38 = 0$ by 2.

(f) If $a, b \in \mathbb{Z}$, not both zero, such that $gcd(a,b) = a\,u + b\,v$, prove that $gcd\,(u, v) = 1$, where $u, v \in \mathbb{Z}$.

(g) Can a null vector be an element of a basis set ? Support your answer.

(h) Find the last two digits in 7^{100}.

(i) If a row echelon matrix R has r non-zero rows, then prove that rank of $R = r$.

(j) If λ be an eigen value of an $n \times n$ matrix A, prove that λ^m is an eigen value of the matrix A^m, where $m \in \mathbb{Z}^+$.

(k) Show that the subspace $U + W$ is the smallest subspace of vector space V containing the subspaces U and W.

(l) For what real values of k is the set

$$S = \{(k,1,1,1), (1,k,1,1), (1,1,k,1), (1,1,1,k)\}$$

linearly independent in vector space $\mathbb{R}^4$?

(m) Let V and W be vector spaces over a field F, and $T: V \to W$ be a linear mapping. Prove that T is injective if and only if $Ker\, T = \{\theta\}$.

(n) Use Euclidean algorithm to find integers u and v satisfying $52u - 91v = 78$.

(o) Use Division algorithm to show that the cube of any integer is of the form $9k$ or $9k \pm 1$, $k \in \mathbb{Z}$.

Group - B

2. Answer any ***four*** questions : 5×4=20

(a) Prove that $\arg z - \arg(-z) = \pm\, \pi$ according as $\arg z > 0$ or $\arg z < 0$.

(b) If a, b, c be positive real numbers and $abc = k^3$, prove that $(1+a)(1+b)(1+c) \geq (1+k)^3$.

(c) Show that the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 + (x-d)^3 = 0$, where a, b, c, d are not all equal, has only one real root.

(d) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then form the equation whose roots are $\alpha + \frac{1}{\alpha}$, $\beta + \frac{1}{\beta}$, $\gamma + \frac{1}{\gamma}$.

P.T.O.

(e) Find a basis and dimension of the subspace S of $\mathbb{R}^3$ defined by
$S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$.

(f) Use the principle of induction to prove that $2.7^n + 3.5^n - 5$ is divisible by 24, $\forall n \in \mathbb{N}$.

Group - C

Answer any *two* questions : 10×2=20

3. (a) If $\alpha = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ and $gcd(n, p) = 1$, then prove that $1 + \alpha^p + \alpha^{2p} + ... + a^{(n-1)p} = 0$.

(b) Prove that in the euqation $f(x) = 0$ with real coefficients, imaginary roots occur in conjugate pairs.

5+5

4. (a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

(b) State Cayley-Hamilton theorem. Using the theorem describe a method of computing A^{-1} when A is a non-singular square matrix. 6+(1+3)

5. (a) If $\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}$ are the eigen vectors corresponding the eigen values 1, 2, 0 of the real square matrix A of order 3, then find A.

(b) Find a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $Im\,T$ is the subspace

$U = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ 5+5

6. (a) For what value of k the planes $x - 4y + 5z = k$, $x - y + 2z = 3$, and $2x + y + z = 0$ intersect in a line ? Find the equations of the line in that case.

(b) If $z = \cos\theta + i\sin\theta$ and $m \in \mathbb{Z}^+$, then show that

$\frac{z^{2m} - 1}{z^{2m} + 1} = i\tan m\theta$. (4+2)+4

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VIDYASAGAR UNIVERSITY

B.Sc. Honours Examination 2021

(CBCS)

1st Semester

MATHEMATICS

PAPER—C1T

CALCULUS , GEOMETRY AND DIFFERENTIAL EQUATION

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *four* questions. 4×12

1. (a) Find the equation of the asymptotes of the curve

$$r^n f_n(\theta) + r^{n-1} f_{n-1}(\theta) + ... + f_0(\theta) = 0$$

(b) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \, Sinnx \, dx$ show that

$2(n - 1) I_n = 1 + (n - 2) I_{n-1}$ and hence deduce

$I_n = \frac{1}{n-1}$ 5+5+2

2. (a) Circles are described on the double ordinates of the parabola $y^2 = 4ax$ as diameters. Prove that the envelope is the parabola $y^2 = 4a(x + a)$.

(b) If $y = \sin\left(m\cos^{-1}\sqrt{x}\right)$ then prove that $\lim_{x\to 0} \frac{y_{n+1}}{y_n} = \frac{4n^2 - m^2}{4n+2}$.

(c) Find a,b,c such that $\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} \to 2 \; as \; x \to 0$. 4+4+4

3. (a) Show that the arc of the upper half of the cardiode $r = a(1 - \cos\theta)$ is bisected at $\theta = \frac{2}{3}\pi$. Find also the perimeter of the curve.

(b) Show that the curve $re^{\theta} = a(1+\theta)$ has no point of inflexion.

(c) Find the asymptotes of the parametric curve $x = \frac{t^2+1}{t^2-1}$ and $y = \frac{t^2}{t-1}$.

4. (a) Show that feet of the normals from the point (α, β, ν) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie on the intersection of the ellipsoid and the cone

$$\frac{\alpha a^2(b^2 - c^2)}{x} + \frac{\beta b^2(c^2 - a^2)}{y} + \frac{\nu c^2\left(a^2 - b^2\right)}{z} = 0.$$

(b) Find the equation of the right circular cylinder whose axis is $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$ and radius is 2. 7+5

5. (a) Prove that cosh(x + y) = coshx coshy + sinhx sinhy.

(b) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of their common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.

(c) Find the polar equation of the normal to the conic $\frac{1}{r}=1+e\cos\theta, e>0$.

2+5+5

6. (a) Find the equation of the generator of the cone $x^2 + y^2 = z^2$ through the point (3, 4, 5).

(b) Given that the asteroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$ is the envelope of the family of ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that a + b = c.

(c) State the existence and uniqueness theorem for the solution of ordinary differential equation. 4+4+4

7. (a) Solve : $x\frac{dy}{dx}-y=x\sqrt{x^2+y^2}$.

(b) If m and n are positive integers, show that

$$\int_a^b (x-a)^m (b-x)^n \, dx = \frac{m!n!}{(m+n+1)!}(b-a)^{m+n+1}$$

(c) Solve y = 2px + y^2p^3 and find the general and singular solutions.

3+4+5

8. (a) Compute the length of the curve $x = 2\cos\theta, y = \sin 2\theta, 0 \le \theta \le \pi$.

(b) Find the points of inflection on the curve $r(\theta^2 - 1) = a\theta^2$

(c) If $I_n = \int_0^1 x^n \tan^{-1} x dx, n$ beine positive integer greater than 2, prove that

$$(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n}$$

3+3+6

Answer any *six* questions. 6×2

9. Find the value of $\lim_{x\to\infty}\left[a_0x^m + a_1x^{m-1} + ... + a_m\right]^{1/x}$, in being a positive integer and $a_0 \ne 0$.

10. Let $I_n = \int_0^1 (lnx)^h\, dx$. Show that $I_n = (-1)^n \underline{|n}$, n being positive integer.

11. The curves y = x^n, y^m = x (m, n > 0) meet at (0, 0) and (1, 1). Find the area between these two curves.

12. Find α if x^α be an integrating factor of $(x - y^2)dx + 2xy\, dy = 0$.

13. Find the curve for which the curvature is zero at every point and which passes through the point (0, 0) where $\frac{dy}{dx} = 3/2$.

14. Solve the differential equation :

$$4x^3 y dx + (x^4 + y^4) dy = 0 \cdot$$

15. Generate a reduction formula for $\int \tan^n x\, dx$, $n \in Z^+$ and n > 1.

16. Find the equations of the straight lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$.

17. Find the asymptote (if any) of the curve y = a log $\left[\sec\left(\frac{x}{a}\right)\right]$.

18. On the ellipse $r(5 - 2\cos\theta) = 21$, find the point with the greatest radius vector.

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VIDYASAGAR UNIVERSITY

B.Sc. Honours Examination 2021

(CBCS)

1st Semester

MATHEMATICS

PAPER—C2T

ALGEBRA

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *four* questions. 4×12

1. (a) If $a_1, a_2, \ldots a_n$ be all positive real numbers and

$S = a_1 + a_2 + \ldots + a_n$;

Prove that $\left(\frac{s-a_1}{n-1}\right)\left(\frac{s-a_2}{n-1}\right)\cdots\left(\frac{s-a_n}{n-1}\right)$

$> a_1 a_2 \ldots a_n$ unless $a_1 = a_2 = \ldots = a_n$

(b) If α, β, γ, δ are the roots of the equation $t^4 + t^2 + 1 = 0$ and n is a positive integer, prove that $\alpha^{2n+1} + \beta^{2n+1} + \gamma^{2n+1} + \delta^{2n+1} = 0$.

(c) Find the relation among the coefficients of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ if its roots be in arithmetic progression. 4+5+3

2. (a) Let C[0, 1] be the set of all real continuous functions on the closed interval [0, 1] and T be a mapping from c[0,1] to R defined by $T(f) = \int_0^1 f(x)dx,\ f \in c[0,1]$. Show that T is a linear transformation.

(b) Let v be a real vector space with a basis $\{\vec{\alpha}_1, \vec{\alpha}_2, .., \vec{\alpha}_n\}$,

Examine if $\{\vec{\alpha}_1 + \vec{\alpha}_2, \vec{\alpha}_2 + \vec{\alpha}_3, .., \vec{\alpha}_n + \vec{\alpha}_1\}$ is also a basis of V.

(c) Find $K \in R$ so that the set S = {(1, 2, 1), (k, 3, 1), (2, k, 0)} is linearly dependent in $1R^3$. 4+5+3

3. (a) Prove that $6 | n(n + 1)(n + 2)$, $n \in \mathbb{Z}$.

(b) Use the theory of congruence to find the remainder when the sum $1^5 + 2^5 + 3^5 + \ldots + 100^5$ is divided by 5. 5+5+2

(c) Find the values of a for which the equation $ax^3 - 6x^2 + 9x - 4 = 0$ may have multiple roots. 5+5+2

4. (a) Find x if the rank of the matrix $\begin{pmatrix} 1 & 3 & -3 & x \\ 2 & 2 & x & -4 \\ 1 & 1-x & 2x+1 & -8-3x \end{pmatrix}$ be 2.

(b) Find the value of λ for which the system of equations

$2x_1 - x_2 + x_3 + x_4 = 1$, $x_1 + 2x_2 - x_3 + 4x_4 = 2$, $x_1 + 7x_2 - 4x_3 + 11x_4 = \lambda$ is solvable.

(c) If $\alpha + \beta + \gamma = 0$, Prove that $\frac{\alpha^5+\beta^5+\gamma^5}{5} = \frac{\alpha^3+\beta^3+\gamma^3}{3} . \frac{\alpha^2+\beta^2+\gamma^2}{2}$

4+4+4

5. (a) If α, β, γ be the roots of the equation $x^3 - 2x^2 + 3x - 1 = 0$,

find the equation whose roots are $\frac{\beta\gamma-\alpha^2}{\beta+\gamma-2\alpha}, \frac{\gamma\alpha-\beta^2}{\gamma+\alpha-2\beta}, \frac{\gamma\beta-\gamma^2}{\alpha+\beta-2\gamma}$

(b) Solve : $(1+x)^{2n} + (1-x)^{2n} = 0$

(c) If $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$, prove that $S_n > \frac{2n}{n+1}$ if $n > 1$. 4+5+3

6. (a) Show that $(2n + 1)^2 \equiv 1 \pmod 8$ for any natural number n.

(b) Use Cayley Hamiltan theorem, to find A^{50} where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

(c) Find the dimension of the subspace $S \cap T$ of $\mathbb{R}^4$ where

$S = \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z+w=0\}$.

$T = \{(x,y,z,w) \in \mathbb{R}^4 : 2x+y-z+w=0\}$. 3+4+5

7. (a) If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A. P where p,q,r are real numbers, prove that $p^2 \geq 3q$.

(b) Find all values of $i^{1/7}$.

(c) Prove that for any two integers U and V > 0,. there exist two unique integers m and n such that

$U = mV + n, \quad o \leq n < V$. 4+4+4

8. (a) If $a \equiv b(\text{mod m})$ and $a \equiv c(\text{mod}\, n)$, prove that $b \equiv c(\text{mod d})$ where d = gcd(m, n).

(b) Find the basis for the column space of the matrix

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

(c) Determine the conditions for which the system of equations

$x + 2y + z = 1$

$2x + y + 3z = b$

$x + ay + 3z = b + 1$

has unique solution, many solutions and no solution.

Answer any *six* questions. 6×2

9. Find the general values of the equation

$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\ldots(\cos n\theta + i\sin n\theta) = -i$, where θ is real.

10. If the equation $x^4 + px^2 + qx + r = 0$ has three equal roots then show that $8p^3 + 27q^2 = 0$.

11. Solve the equations $x + py + p^2z = p^3$, $x + qy + q^2z = q^3$, $x + ry + r^2z = r^3$.

12. Find the equation whose roots are cubes of the roots of the cubic $x^3 + 3x^2 + 2 = 0$.

13. Prove that $n^2 + 2$ is not divisible by 4 for any integer n.

14. Show that the set of all points on the line $y = mx$ forms a sub space of the vector space $\mathbb{R}^2$.

15. Find the number of divisors and their sum of 10800.

16. Find the greatest value of xyz where x, y and z are positive real numbers satisfying $xy + yz + zx = 27$.

17. If A and B be two square invertible matrices, then prove that AB and BA have the same eigen values.

18. Show that eigen values of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$ are all real.

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VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - I

Subject: MATHEMATICS

Paper: C 1-T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Answer any ***three*** from the following questions : 3×20

1. (a) Evaluate the following limits : $\lim_{x\to 0} x ln(\sin x)$ in $(0,\pi)$. 4

(b) Show that the four asymptotes of the curve

$(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^3+2y^3-x^2+3xy-1=0$ cut the curve in eight points which lie on the circle $x^2+y^2=1$. 6

(c) Prove that the envelope of a variable circle whose centre lies on the parabola $y^2=4ax$ and which passes through its vertex is $2ay^2+x(x^2+y^2)=0$ 6

(d) What are the points of inflection of the function $f(x) = 3x^4 - 8x^3$. 4

2. (a) What do you mean by rectillinear asymptotes to a curve ? 4

(b) Find the equation of the envelope of the family of curve represented by equation $x^2 \sin\alpha + y^2 \cos\alpha = a^2$. 4

(c) If $y = \left(\sin^{-1} x\right)^2$ show that $\left(1 - x^2\right) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$. Also find $y_n(0)$. 6

(d) Find the asymptotes of the curve $(x+y)(x-2y)(x-y)^2 + 3xy(x-y) + x^2 + y^2 = 0$. 6

3. (a) If $I_n = \int_0^1 x^n \tan^{-1} x dx$, $n > 2$ then prove that $(n+1) I_n + (n-1) I_{n-2} + \frac{1}{n} = \frac{\pi}{2}$. 4

(b) Determine the length of one arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. 4

(c) Find the reduction formula for $\int \sin^m x Cos^n x dx$ where either m or n or both are negative integers. And hence find $\int \frac{\cos^4 x}{\sin^2 x} dx$. 6

(d) Find the whole length of the loop of the curve $9ay^2 = (x-2a)(x-5a)^2$. 6

4. (a) Find the eccentricity and the vertex of the conic $r = 3\sec^2 \frac{\theta}{2}$. 4

(b) Find the polar equation of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$. 4

(c) A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the locus of the centroid of the triangle ABC is the sphere $9\left(x^2 + y^2 + z^2\right) = 4k^2$. 6

(d) Show that the plane $y+6=0$ intersects the hyperbolic paraboloid $\frac{x^2}{5}-\frac{y^2}{4}=6z$ in parabola. 6

5. (a) For what angle must t he axes be turned to remove the term x^2 from $x^2-4xy+3y^2=0$. 4

(b) Find the centre and the radius of the circle $3x^2+3y^2+3z^2+x-5y-2=0$, $x+y=2$. 4

(c) P is a variable point such that its distance from the xy-plane is always equal to one fourth the square of its distance from the y-axis. Show that the locus of P is a cylinder. 6

(d) Reduce the equation $7x^2+y^2+z^2+16yz+8zx-8xy+2x+4y-40z-14=0$ to the canonical form and find the nature of the conicoid it represents. 6

6. (a) Solve : $(1+y^2)dx-(\tan^{-1}y-x)dy=0$. 4

(b) Find the singular solution of $xp^2-(y-x)p-y=1$. 4

(c) Solve and find the singular solutions of $p^4=4y(xp-2y)^2$. 6

(d) Solve : $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$. 6

UG/1st Sem/MATH(H)/T/19

2019

B.Sc.

1st Semester Examination

MATHEMATICS (Honours)

Paper - C 1-T

Full Marks : 60

Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Unit - I

1. Answer any *three* of the following questions : 3×2=6

(a) If $y = c^{ax} \cos^2 bx$, find $y_n \left(a, b > 0\right)$.

(b) Find the oblique asymptotes of the curve

$$y = \frac{3x}{2} \log\left(e - \frac{1}{3x}\right)$$

[Turn Over]

(c) If $y = x^{n-1} \log x$, then prove that $y_n = \frac{(n-1)!}{x}$

(d) What is reciprocal spiral? Sketch it.

(e) The parabolic path is given by

$$y = x \tan\theta - \frac{x^2}{4h\cos^2\theta}$$

what will be the asymptote of parabolic paths ?

2. Answer any *one* questions : $1 \times 10 = 10$

(a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

(ii) Let $P_n = D^n\left(x^n \log x\right)$.

Prove that $P_n = nP_{n-1} + \lfloor n-1$. Hence show that $P_n = n!\left(\log x + 1 + \frac{1}{2} + \ldots + \frac{1}{n}\right)$. 5

(b) (i) Prove that the envelope of circles whose centres lie on the rectangular hyperbola $xy = c^2$ and which pass through its centre is $\left(x^2 + y^2\right)^2 = 16c^2xy$. 5

(ii) Find the point of inflexion on the curve $\left(\theta^2 - 1\right)r = a\theta^2$. 5

Unit - II

3. Answer any *two* questions : 2×2=4

(a) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin x \, dx, n > 2$. Prove that $2(n-1)I_n = 1 + (n-2)I_{n-1}$.

(b) Find the length of the curve

$$x = e^{\theta} \sin\theta \text{ and } y = e^{\theta} \cos\theta$$

between $\theta = 0$ to $\theta = \frac{\pi}{2}$.

(c) Find the reduction formula for

$$\int \cos^m x \sin(nx) dx.$$

[*Turn Over*]

4. Answer any *two* questions : 2×5=10

(a) Prove that the volume of the solid obtained by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line is $\frac{1}{2}\pi a^3 \left\{ \frac{1}{\sqrt{2}} \log\left(\sqrt{2}+1\right) - \frac{1}{3} \right\}$.

(b) If $I_{m,n} = \int_0^1 x^m (1-x)^n \, dx$,

where m and n are positive integers, then prove that $(m+n+1) I_{m,n} = n I_{m,n-1}$ and deduce that

$$I_{m,n} = \frac{m!n!}{(m+n+1)!}.$$

(c) Evaluate the surface area of the solid generated by revolving the cycloid

$x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ about the line $y = 0$.

Unit - III

5. Answer any *three* questions : 3×2=6

(a) Find the centre and foci of the conic

$$x^2 - 2y^2 - 2x + 8y - 1 = 0$$

(b) Find the equation of the sphere of which the circle $xy + yz + zx = 0, x + y + z = 3$ is a great circle.

(c) Find the condition that the line

$\frac{1}{r} = A\cos\theta + B\sin\theta$ may touch the conic

$\frac{1}{r} = 1 - e\cos\theta.$

(d) For what angle must the axes be turned to remove the term xy from $7x^2 + 4xy + 3y^2$.

(e) Find the equation of cone whose vertex is origin and the base curve is $x^2 + y^2 = 4, z = 2$.

6. Answer any *one* question : 1×5=5

(a) If r be the radius of the circle

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$,

$lx + my + nz = 0$ then prove that

$(r^2 + d)(l^2 + m^2 + n^2) = (mw - nv)^2 + (nu - lw)^2$

$+ (lv - mu)^2$ and find the centre.

[Turn Over]

(b) Show that the feet of the normals from the point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the intersection of the ellipsoid and cone

$$\frac{\alpha a^2\left(b^2-c^2\right)}{x}+\frac{\beta b^2\left(c^2-a^2\right)}{y}+\frac{\gamma c^2\left(a^2-b^2\right)}{z}=0$$

7. Answer any *one* question : 10×1=10

(a) (i) Show that the plane $3x - 2y - z = 0$

cuts the cones $21x^2-4y^2-5z^2=0$ and

$3yz-2zx+2xy=0$

in the same pair of perpendicular lines. 5

(ii) Find the equation of the cylinder, whose generators are parallel to the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and which passes through the conic $z=0, 3x^2+7y^2=12.$ 5

(b) (i) Find the locus of the point of intersection of the perpendicular generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1.$ 4

(ii) Reduce the equation

$$x^2+3y^2+3z^2-2xy-2yz-2zx+1=0$$

to its canonical form and determine the type of quadratic represented by it. 6

Unit - IV

8. Answer any *two* questions : 2×2=4

(a) Find the integrating factor of the differential equation

$$\left(2xy+3x^2y+6y^3\right)dx+\left(x^2+6y^2\right)dy=0$$

(b) Show that the general solution of the equation $\frac{dy}{dx}+Py=Q$ can be written in the form $y=k(u-v)+v$, where k is a constant and u and v are its two particular solutions.

(c) Solve : $\frac{dy}{dx}+y\cos x=xy''$.

9. Answer any *one* question : 1×5=5

(a) The population of a country increases at the rate of proportional to the number of inhabitants. If the population doubles in 30 years, in how many years will it triple?

(b) Solve : $(px^2+y^2)(px+y)=(p+1)^2$

$[u=xy, v=x+y]$